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Generalization of Linked Tensor Decomposition for Group Analysis

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Introduction

background

- Linked tensor decomposition can be used to extract common components, individual components and core tensors simultaneously.
- Linked tensor decomposition can effectively utilize linking/coupling information to improve the identifiability of decomposition.
- Linked tensor decomposition has its advantages in imposing constraints, which can contribute to obtaining more reasonable decomposition results with convincing physiological or pathological interpretations.

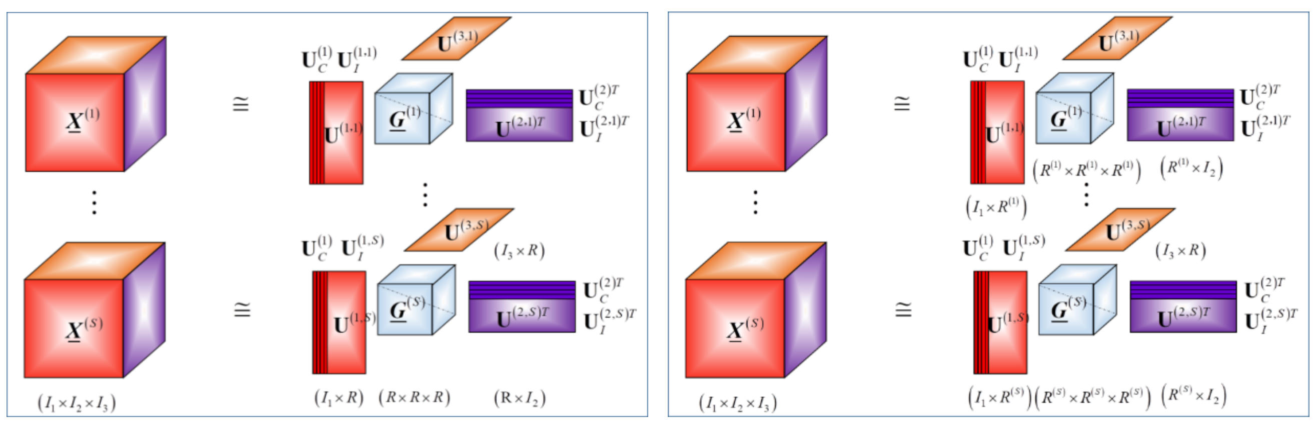
The purpose of the proposed algorithm

To develop a more generalized and flexible model with inconsistent component number of linked tensor decomposition.

Proposed algorithm

Linked tensor decomposition of CP model

- LCPTD model: Consistent component number, as in Fig.1(a).
- Generalization of LCPTD (GLCPTD): Inconsistent component number, as in Fig. 1(b).
- Each factor matrix $\boldsymbol{U}^{(n,s)} = \left[\boldsymbol{U}_C^{(n)}, \boldsymbol{U}_I^{(n,s)}\right]$ consists of two parts: $\boldsymbol{U}_C^{(n)} \in \Re^{I_n \times L_n}, 0 \le L_n \le R$ shared by all tensor blocks with coupling information and $\boldsymbol{U}_I^{(n,s)} \in \Re^{I_n \times (R-L_n)}, 0 \le L_n \le R$ representing individual characteristics of each single tensor block.



(a) LCPTD model (b) GLCPTD model

Fig.1 Conceptual models of dual-coupled LCPTD and Generalized LCPTD

Realization of GLCPTD

- Euclidean Divergence minimization
- Hierarchical Alternating Least Squares (HALS)
- The cost function can be expressed as:

$$\text{minimize} \sum_{s=1}^{S} \left\| \underline{\boldsymbol{X}}^{(s)} - \sum_{r=1}^{R^{(s)}} \lambda_r^{(s)} \boldsymbol{u}_r^{(1,s)} \circ \boldsymbol{u}_r^{(2,s)} \circ \cdots \circ \boldsymbol{u}_r^{(N,s)} \right\|_F^2$$

$$s.t.\ \boldsymbol{u}_r^{(n,1)} = \cdots = \boldsymbol{u}_r^{(n,S)}\ for\ r \le L_n,$$

$$\left\|\boldsymbol{u}_r^{(n,s)}\right\| = 1, n = 1 \cdots N, r = 1 \cdots R^{(s)}, s = 1 \cdots S$$

Experiment

Image reconstruction and denoising

- Yale face database
- 165 gray-scale images of 15 individuals
- 11 images per subject with different facial expressions
- Noise: 5% Salt-and-pepper noise
- Component number: LCPTD (36), GLCPTD (PCA-99.6%)
- Construction of tensors: (1) Face images from the same subject with different expressions; (2) Face images from different subjects with the same expression.

Results

Table 1 shows the averaged PSNRs of reconstructed images under two conditions. Fig. 3 and Fig. 4 depict the original, noisy and reconstructed face images from one subject with four random expressions ('centerlight', 'glasses', 'happy' and 'leftlight') respectively.

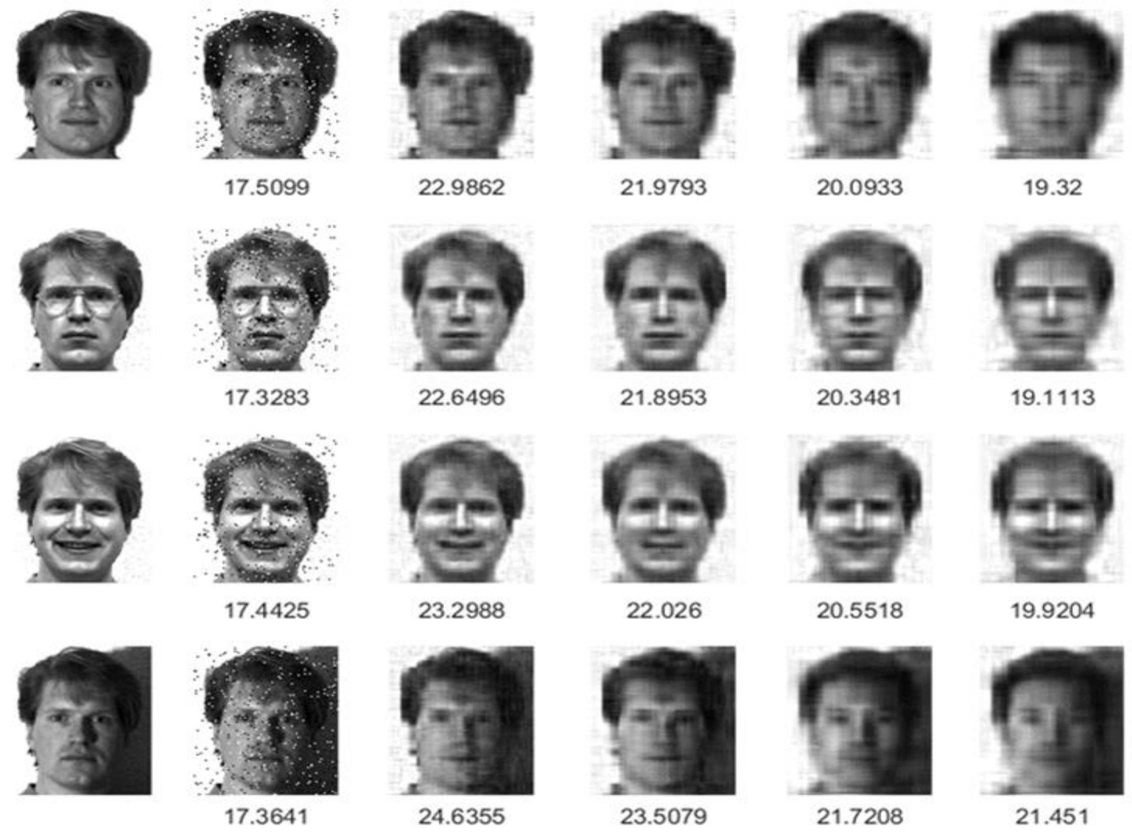


Fig 2. PSNRs of noisy and reconstructed face images. 1st column: original images, 2nd column: noisy images, 3rd column: GLCPTD model of condition I, 4th column: LCPTD model of condition I, 5th column: GLCPTD model of condition II, 6th column: LCPTD model of condition II.

Table 1. Averaged PSNRs (dB) of reconstructed images of condition I&II

	Condition I			*Condition II*		
L_1, L_2	10	20	30	10	20	30
GLCPTD	22.0421	21.5476	20.8134	19.9649	19.6537	19.4444
LCPTD	21.3651	20.7517	19.9021	19.0809	18.8694	18.5321

Conclusion

The main objective of this paper is to develop a generalized and flexible model of linked tensor decomposition which is more suitable for group analysis. The results of image reconstruction and denoising illustrate the superior performance of the newly generalized model. However, the selection of parameter L_n is still an open issue in the current study, which will be one of our future works.

References

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