Fast Double-coupled Nonnegative Tensor Decomposition

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Abstract—**Coupled tensor decomposition has become a popular technique for the simultaneous analysis of multiblock tensors in recent years. To achieve group analysis of multiblock tensors, we propose a fast double-coupled nonnegative Canonical Polyadic decomposition (FDC-NCPD) algorithm. It enables the simultaneous extraction of common components and individual components. In addition, its time-consumption is greatly reduced without compromising the decomposition quality when handling large-scale problems. Simulation results demonstrate the superior performance of the proposed algorithm.**

I. INTRODUCTION

Tensor decomposition has been successfully applied to an ensemble of disciplines including blind source separation, signal processing and neuroscience [1]–[3]. For instance, in EEG data analysis, spatial, temporal and spectral information can be simultaneously considered via tensor decomposition, which in turn provides solutions with convincing physiological or pathological interpretations [3]. However, when it comes to joint analysis of multi-block tensor data, such as multiset or multimodal neurophysiological data fusion [4], conventional methods meet challenges in utilizing coupled information across tensors. Joint analysis of tensors from different samples can potentially reveal underlying structures and inner-relationships among data [5] [6]. Furthermore, joint analysis can take full advantage of prior information to improve the accuracy and stability of solutions [7]. Therefore, increasing recognition of joint analysis makes coupled tensor decomposition more extensively utilized.

Given the ongoing EEG collected multiple subjects under the same stimulus, it is reasonable to expect identical or highly correlated stimuluselicited information among subjects, which can be regarded as a prerequisite for applying coupled tensor decomposition. However, the inner-component similarity among subjects has rarely been considered in previous methods [8] [9]. Meanwhile, the time consumption load would go extremely heavy due to the high-dimensional and nonnegative nature of ongoing EEG data and considering shared information generally exists in spatial and spectral modes, we propose a fast double-coupled nonnegative Canonical Polyadic Decomposition (FDC-NCPD) algorithm. This algorithm is based on linked CP tensor decomposition (LCPTD) model [10] and fast Hierarchical Alternating Least Squares (Fast-HALS) algorithm [11].

II. FDC-NCPD ALGORITHM

To achieve coupled tensor decomposition, squared Euclidean divergence-based cost function is selected as:

$$\min \sum_{s=1}^{S} \left\| \underline{\boldsymbol{X}}^{(s)} - \sum_{r=1}^{R} \boldsymbol{u}_r^{(1,s)} \circ \boldsymbol{u}_r^{(2,s)} \circ \cdots \circ \boldsymbol{u}_r^{(N,s)} \right\|_F^2 \quad (1)$$

$$\text{s.t. } \boldsymbol{u}_r^{(n,1)} = \cdots = \boldsymbol{u}_r^{(n,S)} \text{ for } r \leq L_n,$$

$$\left\| \boldsymbol{u}_r^{(n,s)} \right\| = 1, n = 1 \cdots N-1, r = 1 \cdots R, s = 1 \cdots S.$$

Through HALS algorithm [12] and Fast-HALS algorithm [11], the learning rule of $\boldsymbol{u}_r^{(n,s)}$ can be formulated as follows:

$$\boldsymbol{u}_r^{(n,s)} = \begin{cases} \left[\sum_s \zeta_r^{(n,s)} \right] / \sum_s \gamma_r^{(n,s)}, r \leq L_n, \\ \zeta_r^{(n,s)} / \gamma_r^{(n,s)}, \qquad r > L_n, \end{cases} \quad (2)$$

where the scaling coefficients $\gamma_r^{(n,s)}$ can be calculated as:

$$\gamma_r^{(n,s)} = \begin{cases} \boldsymbol{u}_r^{(N,s)T} \boldsymbol{u}_r^{(N,s)}, & n \neq N. \\ 1, & n = N. \end{cases} \quad (3)$$

and

$$\zeta_r^{(n,s)} = \left[\underline{\boldsymbol{X}}_{(n)}^{(s)} \{ \boldsymbol{U}^{(s)} \}^{\odot -n} \right]_r - \boldsymbol{U}^{(n,s)} \left[\xi_{(n)}^{(s)} \right]_r + \gamma_r^{(n,s)} \boldsymbol{u}_r^{(n,s)}, \quad (4)$$

where $\xi_{(n)}^{(s)} = (\boldsymbol{U}^{(s)T} \boldsymbol{U}^{(s)})^{\circledast} \oslash (\boldsymbol{U}^{(n,s)T} \boldsymbol{U}^{(n,s)})$. '$\circledast$' and '$\oslash$' are denoted as element-wise multiplication and division. In order to obtain the nonnegative components, a simple "half-rectifying" nonlinear projection is applied as $\boldsymbol{u}_r^{(n,s)} \leftarrow \| \boldsymbol{u}_r^{(n,s)} \|_+$ after (2). These R stages are updated alternatively one after another until convergence.

III. EXPERIMENTS AND RESULTS

Exp1. Validation of synthetic data. Fig. 1 illustrates execution time against the dimensionality of tensors averaged over 30 runs. SNR = 20 dB, $R = 4n$, $L_1 = L_2 = 2n$, $S = 10$. Fig. 2 illustrates averagedd decomposition performance of four algorithms [10]–[12] from 20 runs under SNRs from -5 dB to 20 dB. $I_1 = 40, I_2 = 50, I_3 = 60$, $R = 30$, $L_1 = L_2 = 20$ and $S = 10$. FDC-NCPD algorithm could greatly reduce the execution time while keeping excellent decomposition quality. This experiment also verified that joint/coupled analysis can effectively improve the decomposition accuracy.

Exp2. Application of ongoing EEG data. We apply the FDC-NCPD algorithm to ongoing EEG data, collected from 14 subjects while listening to an 8.5-minute long tango music. The details of data collection, data preprocessing and related infromation can be found in [9]. Through short-time Fourier transform (STFT), 14 third-order tensors are formulated with size of $64 \times 146 \times 510$ (64 spatial channels, 146 frequency bins (1∼30Hz) and 510 temporal samples from EEG data of each subject). The results in Fig. 3 and Fig. 4 illustrate FDC-NCPD algorithm can efficiently and reliably explore the underlying brain activities under naturalistic and continuous musical stimulus.

IV. CONCLUSION

We introduced the Fast-HALS algorithm to LCPTD model and proposed the FDC-NCPD algorithm, in which the common components, individual components can be extracted simultaneously. Simulation experiments of synthetic and real-world data verified the performance of proposed algorithm.
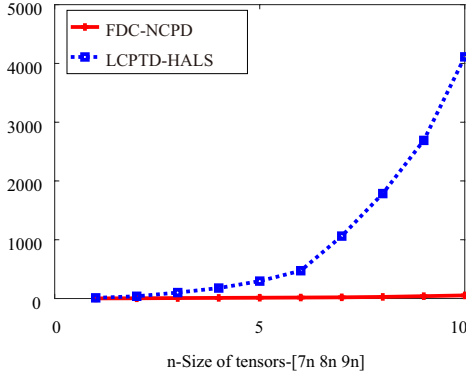
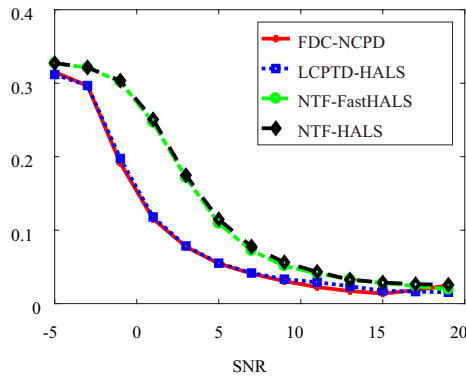Fig. 1. Averaged execution time versus dimensionality of tensors

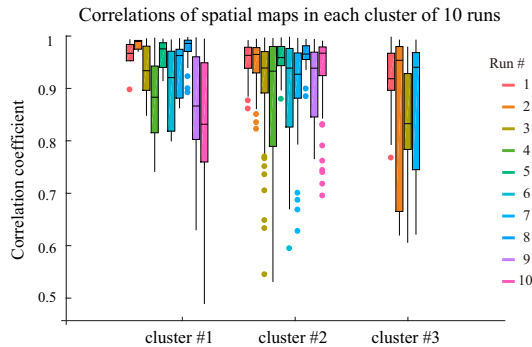Fig. 2. Averaged PI performance versus SNR

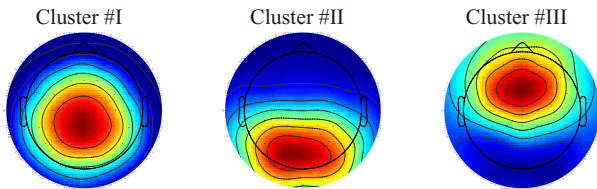Fig. 3. Correlation coefficients of internal components of clusters in 10 runs

Fig. 4. Averaged topographies of interest clusters from 10 runs

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